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Relaxation Processes and Black Holes in Star Clusters and Galactic Nuclei

Marc Hemsendorf³, Emil Khalisi¹, Chingis Omarov^{1,2}, and Rainer Spurzem¹

¹ Astronomisches Rechen-Institut, Mönchhofstrasse 12-14, 69120 Heidelberg, Germany
E-mail: {khalisi, omarov, spurzem}@ari.uni-heidelberg.de

² Fessenkov Astrophysical Institute, 480068 Alma-Ata, Kazakhstan

³ Dept. of Physics and Astronomy, Rutgers State Univ. of New Jersey USA, 136 Frelinghuysen Road, Piscataway, NJ 08854-8019, USA
E-mail: marchems@physics.rutgers.edu

We use high-precision direct N -body integration to study questions of the thermodynamic behaviour of dense stellar systems and of the state of dense stellar systems interacting with binary black holes in galactic nuclei. Thermodynamical processes examined include mass segregation and equipartition processes and stellar orbits in galactic nuclei with thick accretion disks.

1 Introduction

In this paper we describe results of three ongoing intertwined subprojects. They are all based on the numerical simulation of dense stellar systems (systems of gravitating point masses) by direct accurate orbit integrations. We use the direct parallel N -body code NBODY6++³⁵. The first subproject has been followed already in the previous computing periods and is published meanwhile^{13,12}. We study the interaction of two black holes, shrinking by dynamical friction towards each other, forming a tightly bound binary, which is expected in many galactic nuclei as a result of merging processes. The interested is referred to the two cited papers; ongoing studies are now turning to different mass ratios of black holes and variation of density profiles in the merging galaxies. A second subproject (studies of mass segregation and equipartition) has been completed in the PhD thesis of Dr. Emil Khalisi¹⁸ has been completed and can be obtained (in English) by

<http://www.ub.uni-heidelberg.de/archiv/3096>

and a publication is in preparation¹⁹. The third subproject (orbit studies of stellar orbits in galactic centres with thick accretion disks) is mainly performed by Dr. Chingis Omarov, who is working in our institute as a DAAD fellow; this work is related to SFB439 “Galaxies in the Young Universe” at the Univ. of Heidelberg.

For all these and other related projects the highly accurate determination of gravitational and other forces acting on particles on their orbits is very important. There are three main problems which require such accuracy:

- the correct modelling of relaxation processes (e.g. mass and energy transport due to small angle star star gravitative encounters) determines the rates with which stars in a dense stellar cluster are transported into a massive accretion disk and also further down to the massive black hole. Gravity is not shielded at something like a Debye sphere so all interactions (even those with large impact parameters) must be followed by high precision (i.e. relative energy error of 10^{-5} over hundred crossing times, to make a specific example).

- the larger the star clusters are the longer the relaxation time becomes as compared to the orbital or crossing time. Therefore simulations consist of tens of billions of individual steps and this is another reason why any secular force errors have to be kept at the lowest technically and numerically possible level.
- there is an intrinsically large dynamic range even at any given moment, for example planetary orbits have time scales of a few days in the worst case, while the relaxation in a star cluster may be up to a few Gigayears. That's around ten orders of magnitude, and another reason why none of the approximative methods used elsewhere in N-body dynamics (TREE, series expansions, mesh based Fourier transformation) can be used.

The only massively parallel code able to fulfil all requirements from the three items above is NBODY6++³⁵, which uses an Ahmad-Cohen neighbour scheme, direct force computations (asymptotically the algorithm scales with N^2 , but individually blocked time steps and regularisations of close encounters and bound subsystems (such as the planetary systems). The reader interested in more details about this family (very often called Aarseth-codes) is referred to the above cited paper and two more overview papers by Aarseth^{1,2}.

For any particle number larger than a few 10^4 significant simulations require the use of massively parallel computers with extremely fast communication (of the order of Gbit/s simultaneously between many pairs of nodes). In the figure we can see how the use of the CRAY T3E turned the N^2 scaling of our algorithm down into a linear dependence. We can also see that a recent PC cluster obtained at our institute for SFB439 (20 P4 CPU's, 2.2 Ghz, Myrinet 2000) is for less than 50k particles meanwhile faster than the T3E. However, we need particle numbers much larger than 50k in the future, up to one million, in particular for all problems related to galactic nuclei. Here we will still be requiring the large scale parallel computing facilities of HLRS, in particular we are looking forward what will be the performance of the new generation of supercomputers (IBM, NEC, Hitachi) with our codes. Similarly, the Japanese GRAPE-6 special purpose computers seem to beat all other machines by a large factor. However, it is only able to outplay this performance for pure gravitational problems. Any additional complexities such as interactions with gas or planetary or binary system will make it impossible to use it with that performance. Rather, the general purpose supercomputers can be used much more efficiently here (that includes also the PC clusters).

2 Limits of Mass Segregation in Two-Component Models

Main Author of this section: E. Khalisi

2.1 Introduction

During the past two decades the theoretical and numerical study of the relaxation driven evolution of star clusters has seen an enormous progress. One of the important reasons was a dramatic increase in the capabilities of fast general and special purpose computers such as GRAPE^{38,27}, but also direct N -body modelling software for them^{35,1,2}. In a large number of studies the evolution of stellar clusters has been studied and the validity of simplified theoretical models assessed, including many astrophysical effects such as stellar

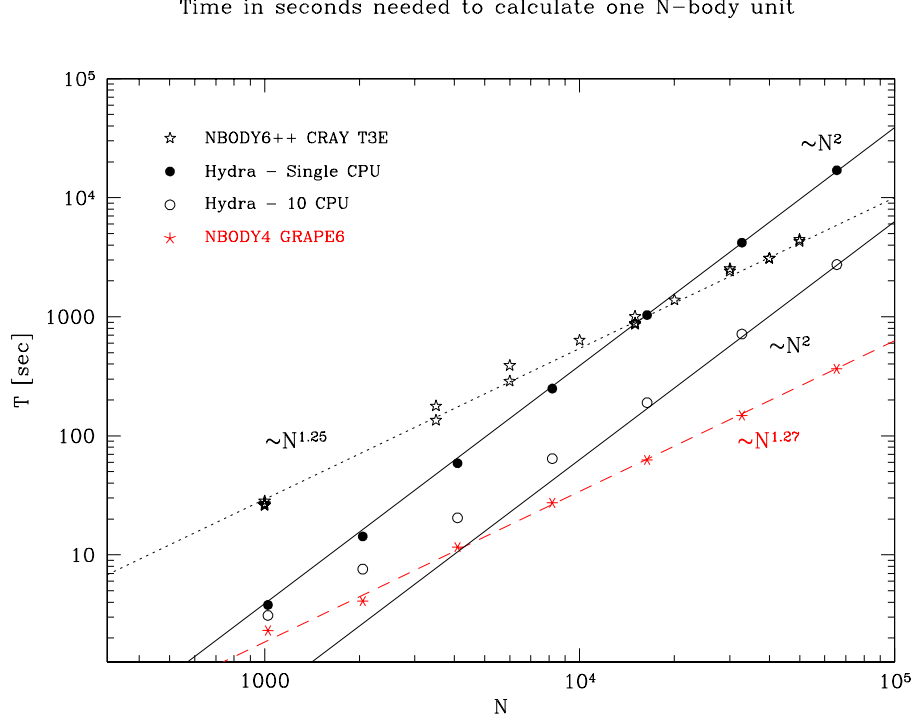


Figure 1. Wallclock time for one third of a crossing time used on different hardware for our direct NBODY codes.

evolution, realistic tidal fields, primordial binaries, mergers and collisions (compare the reviews^{26,36}).

While it is still impossible to directly model globular star clusters or galactic nuclei with realistic particle numbers, there is another class of clusters which have just a particle number feasible for a direct N -body study: young open star clusters with relaxation effects going on. They have been out of focus for the pure dynamicists in many cases, although new deep infrared observations of the Trapezium Cluster in Orion^{25,14,15} allow a detailed measurement of the kinematics and distribution of their stars. The results show a considerable mass segregation of heavy stars to the centre; according to¹⁵ the average mass of stars within the core radius of the cluster (0.205 pc) is three times larger than the average mass within 2 pc. The cluster is less than 1 Myr old.

Collisional star cluster evolution naturally leads to a segregation of heavy masses in the core due to two-body relaxation, an effect already known for long time³³. If the amount

of heavy stars is relatively minor, an accelerated core collapse takes place, including all stars, but leading to an increase of the average stellar mass in the centre. Systems with a large amount of heavy masses tend to separate the heavy masses out completely, the latter undergoing a dynamically decoupled core collapse on their own. Such distinction has already been made in Spitzer's³³ mass segregation instability. If in a system with only a few thousand or ten thousand stars, it could be more appropriate to describe the process as dynamical friction, which brings the few most heavy stars to the centre.

Surprisingly, there has been only little quantitative study of mass segregation in the pre-collapse phase of star clusters, probably because more interest was prevailing in the post-collapse evolution of globulars. Bonnell & Davies⁴ delivered a study using ensemble averaged results of clusters of $N = 150$ to $N = 1500$. Their conclusion was that the mass segregation time scale is not sufficiently small to account for the observed segregation in young clusters. However, they were using an N -body code with softening, thus cutting off all close encounters and binary formation, which affect the cluster evolution strongly in late collapse. Also, the maximum particle number of 1500 does not allow statistically secure conclusions for clusters in which the most massive stars are 20 or 30 times more massive than the average star, because there is only a very small number of massive stars.

One variant of star formation theory in clusters by Murray & Lin²⁸ predicts the formation of the most massive stars already near the cluster centre, since their proto-stellar clouds require many dissipative mergers with low-mass cloudlets for the growth. They are to happen, of course, most likely in the dense regions. Such newly formed clusters will remain gravitationally bound with the massive stars remaining preferentially in the inner parts of the cluster. On the other side, Podsiadlowski & Price (1992) draw a scheme in which massive stars might also form in some cold gaseous clumps in the outskirts of a star-forming region. These individual massive stars sink to the centre (possibly while still forming), and are likely to affect the formation of other stars, e.g. by heating or disrupting their gas cloud. It is only the later phases of star formation that will be dominated by a sequential formation of massive stars surrounded by previously formed low-mass stars. However, the different results are conflicting.

Regarding the present unclear situation our approach is rather to reduce the complexity of the system, but to study the physical mechanism of mass segregation in young star clusters more systematically. The results will be related to the above papers and observations. As a completely new study, we investigate the effect of initial segregation and “anti”-segregation on the evolution of the system. We explore the range of the segregation process and compare its saturation according to the different initial configurations. Comparisons with other simplified models are given.

2.2 The Models

All simulations start as Plummer spheres, which appear to be very similar to a King $W_0 = 6$ model in its core collapse parameters (Quinlan 1996). They start from a global virial equilibrium, and the particles are treated as point masses, i.e. without softening. An external tidal field, primordial binaries, and stellar evolution are neglected here. Our model parameters were chosen to closely match the observational results of Hillenbrand & Hartmann¹⁵ for the Orion cluster: 5000 stars, in two mass components (subscripts 1 for the light stars, and 2 for the heavies), with $q := M_2/M_{\text{tot}} = 0.26$, and $\mu = m_2/m_1 = 20$, where

M_1, M_2 are the total masses of the two components, and m_1, m_2 are the individual stellar masses, respectively. The total number of particles is $N = N_1 + N_2 = 4914 + 86 = 5000$.

The spatial distribution of each mass m_1 or m_2 is chosen randomly to initialize a different setup of positions and velocities at the start of the simulation. These physically equivalent runs were repeated 10 times and averaged into a mean model — we shall call this the “RND”-model. The goal of such “ensemble averaging” is to increase the statistical significance of the global output data⁸.

Additionally, we tested two exotic setup configurations by placing the heavy masses m_2 artificially at the two extreme parts: First, all of them in the inner regions such that the two mass components would already be maximum segregated and no further process of stratification expected — those models will be called “INS”; second, in the outside regions foothills of the Plummer density profile, in order to construct a most possible “anti-segregation” — these models are named “OUT”. Though unrealistic, such extreme cases are of special interest for investigating the full range of the segregation process. The limits give us for example the shortest and longest time scale. As before, both kind of models are ensemble averaged over 10 individual runs.

Besides these models, we also varied the parameter of the mass ratios in the following steps: $\mu = 1.25, 1.5, 2.0, 3.0, 5.0, 10.0, 20.0, 40.0, 75.0, 100.0$. They serve as a check how the course of the core collapse times changes with respect to Spitzer’s accelerated evolution.

2.3 Selected Results

To discuss the segregation of masses, we have divided the system in spherical mass shells containing 1, 2, 5, 10, 20, 30, 40, 50, 75, 90, 100% of the total mass, respectively. The changes of the mean mass inside such a shell is plotted in Figure 2. The innermost shells quickly gather the heavy masses and undergo a core collapse. In the post-collapse phase, the mean mass remains in the centre is rather constant indicating that the global segregation has come to an end. The intermediate shells still [anhäufen] the heavy masses, though somewhat the progress has slowed down in the core bounce. The degree of segregation in the shells can be understood as the mean mass achieved with regard to the maximum possible mass (i.e. the case if a shell would consist of the heavy particles only), $\langle m \rangle / m_2$. For this model the degree in the innermost shell reaches about $11M_\odot / 20M_\odot \approx 0.55$, the lower next shell about 0.4, etc.

The segregation of the heavy stars is to be compared with the range of “possible” segregation processes. In order to exploit a minimum and maximum time scale, we performed two more cases of the same model in which the heavy stars have been artificially placed either in the inside regions such that a pre-segregated state is given (we call it an INS-model), or in the outer halo for constructing an “anti-segregated” state (OUT-model). Though unrealistic, such extreme cases disclose the width of the segregation time scale. Figure 3 illustrates the evolution of the mean mass in both models: In the left panel, the cluster starts from the INS-configuration: The 1%-Lagrangian shell has got a mean mass that is μ -times higher than the outer layers. The light stars penetrate immediately after the start into the innermost shell and reduce $\langle m \rangle$, as seen at the very left margin. The sphere of heavy bodies expands a little and raises the mean mass in the outer spheres (lower curves). After the short settling period of some few crossing times, the model turns into a configuration as in the RND-model.

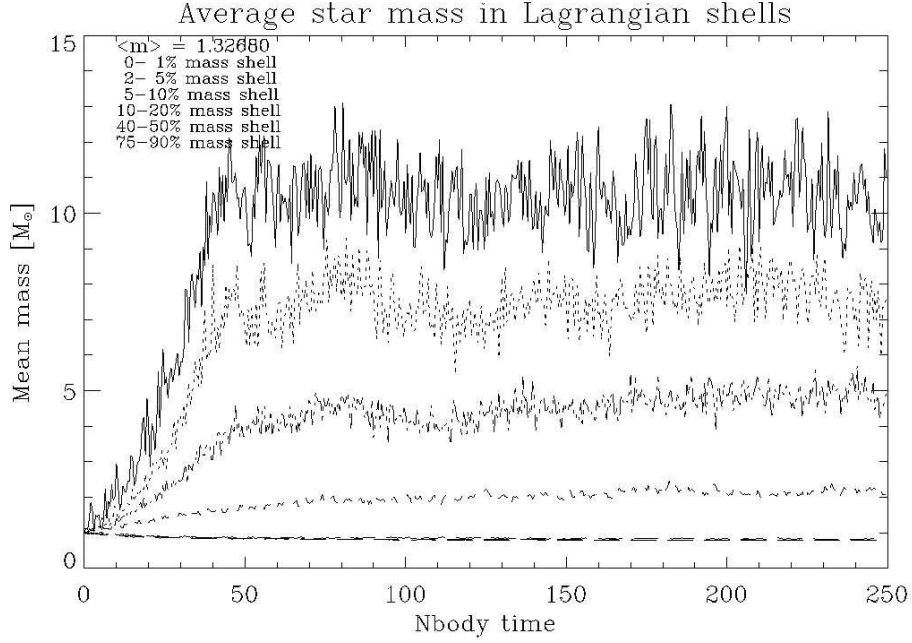


Figure 2. Evolution of the mean mass within Lagrangian shells of the two-mass model RND-G. The raw data of 10 independent runs was averaged into one single ensemble-model.

In the right panel of Figure 3, it is visible how slowly the heavy masses sink from the outskirts to the centre; the decreasing dashed line is the mean mass of the 75–95% shell. Its slow decline indicates that fair number of heavies remains in the halo for a long time before they start falling inwards. Their orbital velocity is also small, and their motion starts from a rather inert state. The collapse is performed a by those few heavy bodies that by chance moved quicker to the centre. As seen from the lower $\langle m \rangle$ in the innermost shell, the number of the heavies being present at the time of the collapse is smaller than in the INS- and RND-models.

In addition to this model that matches the parameters of the Orion Cluster, we exploited the core collapse times for other relative masses m_2/m_1 , i.e. for a wide range of μ 's from 1 to 75. The maximum value might correspond to an imaginary cluster with two different masses like $0.08M_\odot$ and $6.0M_\odot$, but a constant fraction of 26% heavies. The core collapse times and their ranges are summarized in Figure 4. The triangles indicate the values from the random setup and the dashed lines are the INS- and OUT-models.

From these preliminary results we can conclude that the time scale for the heavy masses to reach the cluster centre, where they are observed today in young clusters, is not much changed, even if one starts with an initial segregation of heavy masses inwards (model INS). If, however, heavy masses are predominantly outside (model OUT) the time is much longer for them to get into the centre. So we can constrain possible models of star formation by looking at the dynamical evolution of the cluster. More details will be published soon¹⁹. In future work we want to study quantitatively how considerable fractions of initial

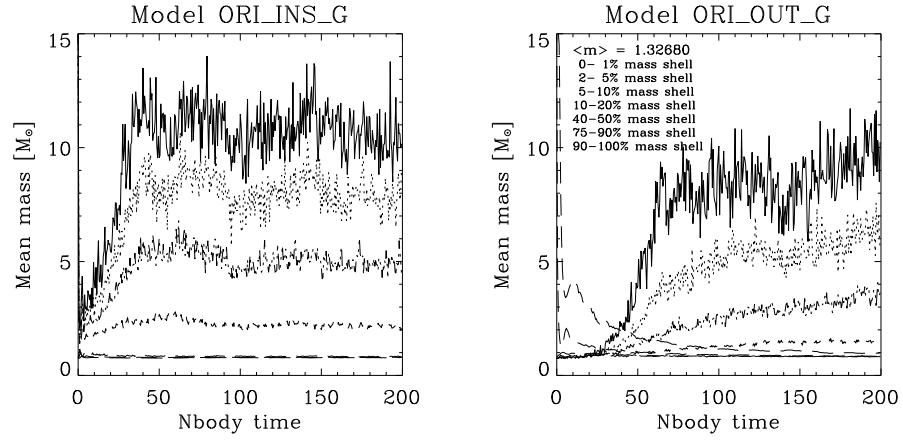


Figure 3. Comparison of the mean mass between Lagrangian radii for Models G-INS (*left*) and G-OUT (*right*).

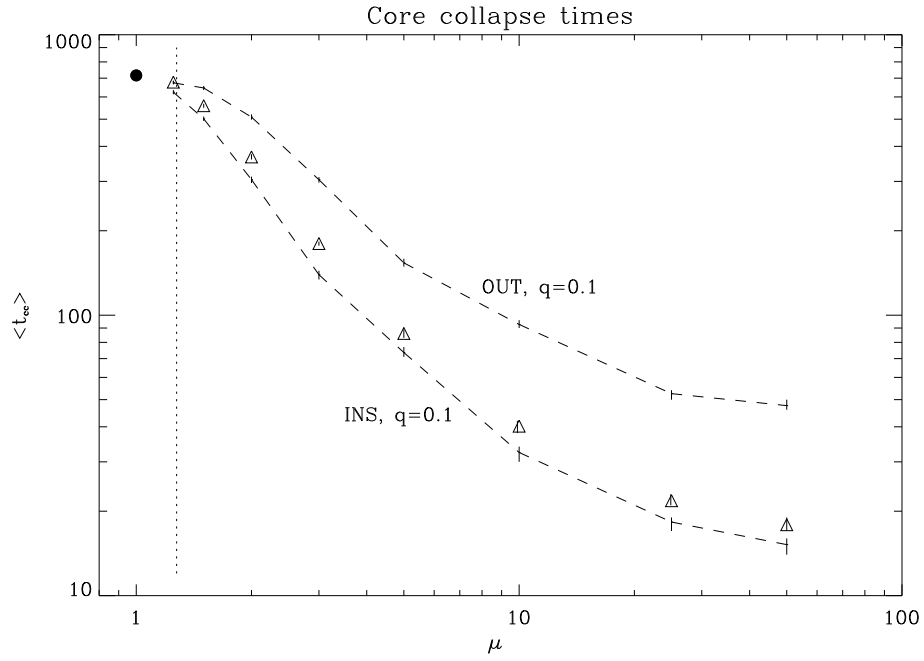


Figure 4. Core collapse times for models with $N = 5000$ of the random configuration (triangles) and the extreme models INS and OUT, in which the heavy stars were placed in the center or to the outskirts (dashed lines). The filled circle represents the value of the equal-mass system. The small error bars are also indicated.

(primordial) binaries in star clusters influence the mass segregation processes, in order to test the recent results obtained with a hybrid Monte Carlo model^{9,10} and other Monte Carlo models⁷.

3 Stellar Orbits in Galactic Centres with Accretion Disks

Main Author of this section: Ch. Omarov

The main physical subsystems of active galactic nuclei (hereafter AGNs), containing most of the mass of AGN are the compact stellar cluster (CSC) and the massive black hole (MBH) at the center of the cluster. The last one is usually surrounded by an accretion disk (AD), providing most of the luminosity of bright AGNs. Apparently, the dissipative interaction of the CSC with the AD determines the stellar dynamics in the central part of the cluster and, consequently, the AGN evolution. Nevertheless, the task of the evolution taking into account both stellar two-body gravitation interaction and the star-disk interaction still is not solved, though many works were devoted to the stellar dynamics in central parts of AGNs. The interaction of the compact stellar cluster with a massive central object in the forms of super- star and MBH was considered by Vilkoviski, Hara, and Hills^{40,16,11}. The evolution of the dense non-rotating stellar cluster was studied by Spitzer, Saslaw, Bisnovaty-Kogan among others^{34,5}, and the evolution of the gas sphere was considered by Langbein and collaborators²⁴. Stellar interactions with accretion disks were as well considered by Vilkoviskij and Syer^{41,42,39}. More detailed investigations of the stellar orbits, crossing accretion disks were presented in the works by Artymowicz, Karas, Vokrouhlicky^{3,44,37}. Finally Rauch^{30,31} considered and numerically calculated the cases when non-elastic star-disk interactions or pure star-star collisions dominate, and Vilkoviskij and Cherny⁴³ calculated an analytical model of the joint action of the star-disk and star-star interactions.

This ongoing projects is examining by direct N -body simulation the problem of the interaction of compact stellar cluster with the accretion disk in active galactic nuclei. We have accomplished the inclusion of a standard ram pressure force into the Hermite scheme of the N -body code NBODY6++. This is the first time that such high precision direct N -body simulation considers non-gravitational forces. However, our disk is yet stationary, with a Keplerian rotation velocity. We include as free parameters of order unity the interaction coefficient of the ram pressure force and the Keplerian velocity at the inner edge of the disk.

Vilkoviskij & Cherny⁴³ have compared the star-star two-body interactions with the star-disk interactions and concluded that the last is more strong in the inner parts of the ADs. The rate of the change of a star energy E due to the two-body star-star interaction is

$$dE^{(ss)}/dt = 4\pi G^2 M_s^3 V_s^{-1} \ln \Lambda, \quad (1)$$

where G is the gravitation constant, M_s and V_s — mass and velocity of the star, $\ln \Lambda$ - the Coulomb logarithm.

It can be compared with the rate of the change of a star energy due to the non-elastic star-disk interaction:

$$dE^{(sd)}/dt = \pi q(i) R_s^2 \Sigma_d V_s^2 / T, \quad (2)$$

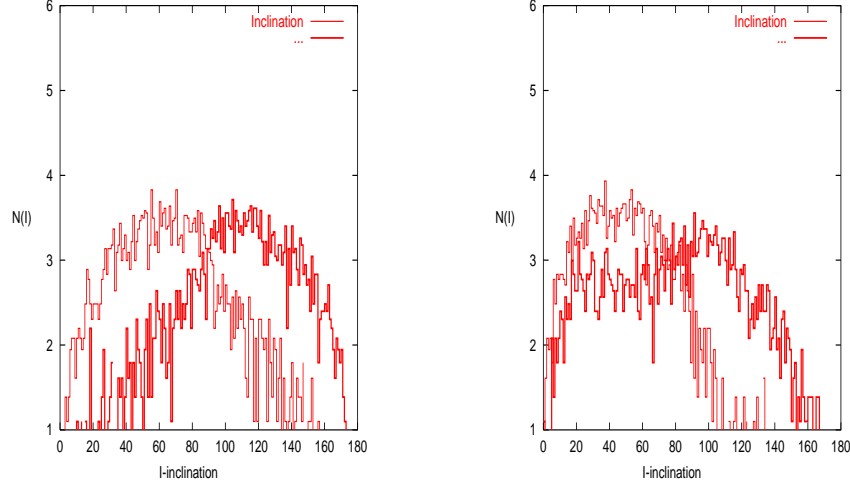


Figure 5. Distribution of inclinations of stellar orbits in a galactic nucleus with a massive accretion disk. Thin curves co-rotating stars, thick curves counter-rotating stars. Left panel initial model. The original distribution should be like a sine function (if both components are added). Right panel after a few crossing times, where the interaction with the disk has depleted the counter-rotating orbit family.

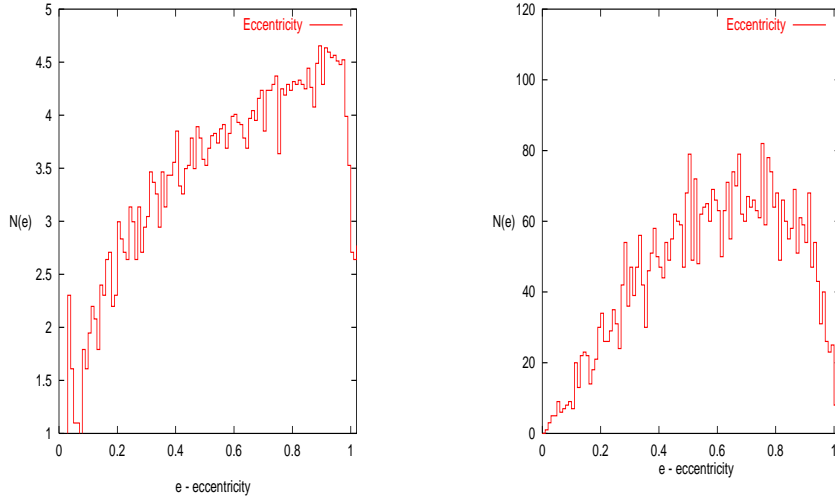


Figure 6. As in Fig.5, but here showing the eccentricities of all the orbits, not separated for their sense of motion. We can see that high eccentricities are depleted.

where $q(i) < 1$ is dissipation parameter, depending on the inclination angle i , R_s – the radius of the star, Σ_d – the surface mass density of the accretion disk, and T – the orbital period of the star.

One can see that in the close-to-Kepler potential of the BH in the inner part of the AD, $dE^{(sd)}/dt > dE^{(ss)}/dt$ due to the different velocity dependence of the both values. On

the other hand, the star-disk interaction leads to inclination of the stellar orbits to the plane of the disk, which diminish $dE^{(sd)}/dt$. Without the star-star scattering, the inclination angle and the energy dissipation could go to zero, but if the stellar density is large enough, the scattering will increase the inclination angle. As a result, both $dE^{(sd)}/dt$ and $dE^{(ss)}/dt$ are keeping in some equilibrium, regulating the inclination angle and the inflow of the stars to the centrum of the AD.

A simple model for the stellar distribution and the inflow was calculated in the work by Vilkoviskij and Cherny (2002), and it was shown that stars can influence the AGN variability and the emission broad line regions properties.

As first examples of our results we show the initial and final (after a few orbital times) inclinations and eccentricities of 5000 stars in sample calculation. One can see how the interaction with the disk creates a lack of high inclinations and also influences co- and counterrotating stellar orbits in a different way. This will affect the global shape and angular momentum distribution of the system. With more simulation data we will be able to predict data of the central accretion disk (not resolved by direct observations) from the stellar kinematics much further outside where it can be observed.

These data will be completed using different particle numbers in the present computing period, and a refereed publication will be completed describing results from a larger parameter study using different particle numbers, if the visit of Dr. Omarov can be prolonged until Sep. 30, 2003.

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